

$$\left(x - \frac{u^2 \sin^2 \alpha}{2g}\right)^2 = -\frac{2u^2 \cos^2 \alpha}{g} \left[y - \frac{u^2 \sin^2 \alpha}{2g}\right]$$

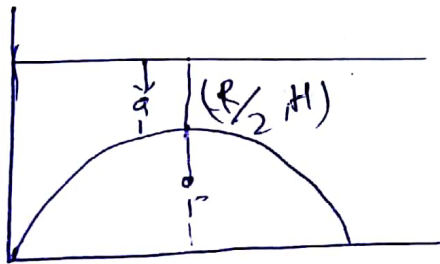
$$\boxed{\left(x - \frac{R}{2}\right)^2 = -4 \frac{u^2 \cos^2 \alpha}{2g} (y - H)}$$

Vertex :-

$$x = \frac{u^2 \sin^2 \alpha}{g}$$

$$y = \frac{u^2 \sin^2 \alpha}{2g}$$

Path of vertex $\rightarrow$ Ellipse



Here $a = \frac{u^2 \cos^2 \alpha}{2g}$

eqn of directrix

$$y = H + a$$

$$y = \frac{u^2}{2g} (\sin^2 \alpha + \cos^2 \alpha)$$

$$y = H + a = \frac{R_{\max}}{2}$$

$$\leftarrow y = \frac{u^2}{2g}$$

Locus of focus

$$x = \frac{R}{2} = \frac{u^2 \sin^2 \alpha}{2g}$$

$$y = H - a = \frac{u^2}{2g} (\sin^2 \alpha - \cos^2 \alpha)$$

$$= -\frac{u^2 \cos^2 \alpha}{2g}$$

$$F = \left(\frac{u^2 \sin^2 \alpha}{2g}, -\frac{u^2 \cos^2 \alpha}{2g} \right)$$

F eqn of path of focus

$$x^2 + y^2 = \left(\frac{u^2}{2g}\right)^2$$

circular track

$$\text{Radius} = \frac{R_{\max}}{2}$$

Q/

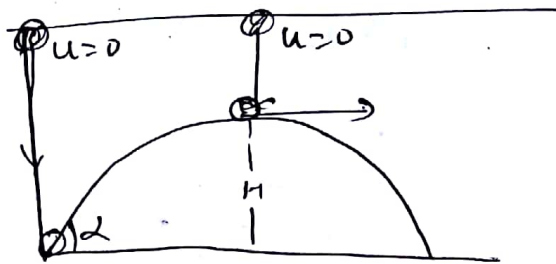
Angle of ProjectionLocus of focus

- (a) $\angle < 45^\circ$ → (P) focus on the plane
 (b) $\angle = 45^\circ$ → (Q) focus below plane
 (c) $\angle > 45^\circ$ → (Y) focus above plane

Q/

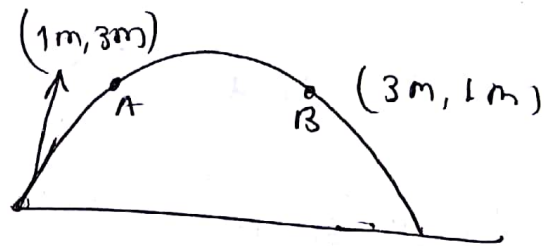
Speed of ball released from direction

at A = $v = \sqrt{2g \frac{u^2}{2g}}$
 $v = u$



At A =

If projected body passes through Pt. A and Pt. B then find its range.

Pt. A

$$3 = ax^2 - bx \quad x=1$$

$$3 = a - b \quad \text{--- (I)}$$

Pt. B

$$1 = ax^2 - bx \quad x=3$$

$$1 = 9a - 3b \quad \text{--- (II)}$$

$$\frac{a-b}{9a-3b} = \frac{3}{1}$$

$$a-b = 27a-27b$$

$$8a = 26b$$

$$\frac{a}{b} = \frac{26}{8} \Rightarrow \frac{13}{4} \text{ m Ans.}$$

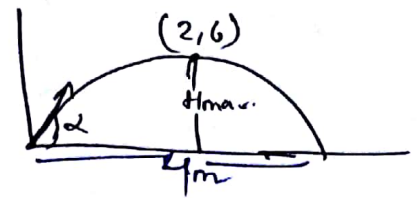
Q// find angle α

$$H = \frac{a^2}{4b} = \frac{a}{b} \cdot \frac{a}{4}$$

$$H = R \cdot \frac{a}{4}$$

$$\frac{4H}{R} = a \Rightarrow \tan \alpha = \frac{4H}{R} \Rightarrow \frac{4 \times 6}{4} = 6 \quad \boxed{6}$$

$$\tan \alpha = 6, \quad \boxed{\alpha = \tan^{-1} 6}$$



Q//

what is $x_{\min}$.

$$y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha)$$

$$0 = 10 \tan \alpha - \frac{9 \times 100}{2 \times 100} (1 + \tan^2 \alpha)$$

$$10 \tan \alpha = \frac{9}{4} + \frac{9}{4} \tan^2 \alpha$$

$$5 \tan^2 \alpha - 40 \tan \alpha + 9 = 0$$

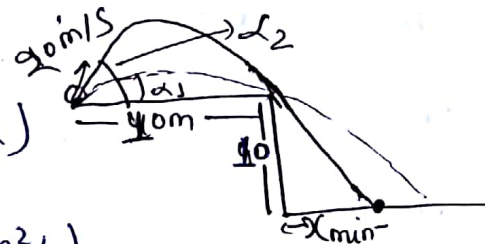
$$\tan^2 \alpha - 8 \tan \alpha + 1 = 0$$

α_2

$$-10 = x \tan \alpha_2 - \frac{9x^2}{2 \cdot (20)^2} [(1 + (\tan \alpha_2)^2)]$$

$$x = ?$$

$$x_{\min} = x - 10$$

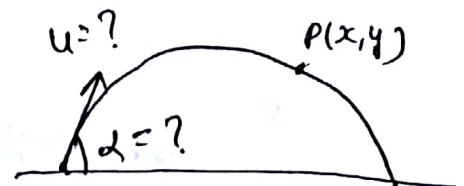


3//

Required velocity & angle of projection so that projected body can pass through point $P(x, y)$

eqⁿ of path

$$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha$$



for given α , y is const.

$$\frac{dy}{dx} = 0$$

$$x \sec^2 \alpha - \frac{gx^2}{2u^2} \cdot 2 \sec^2 \alpha \tan \alpha = 0$$

$$\boxed{\tan \alpha = \frac{u^2}{gx}}$$

$$y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha)$$

$$y = \frac{xu^2}{gx} - \frac{gx^2}{2u^2} \left(1 + \left(\frac{u^2}{gx} \right)^2 \right)$$

$$y = \frac{u^2}{g} - \frac{gx^2}{2u^2} \left(1 + \left(\frac{u^2}{gx} \right)^2 \right)$$

$$y = \frac{u^2}{g} - \frac{gx^2}{2u^2} \left(\frac{g^2x^2 + u^4}{g^2x^2} \right)$$

$$y = \frac{u^2}{2g} - \frac{gx^2}{2u^2} - \frac{gx^2 u^4}{2u^2 g^2 x^2}$$



$$\text{max. } y = \frac{u^4 - g^2x^2}{2gu^2}$$

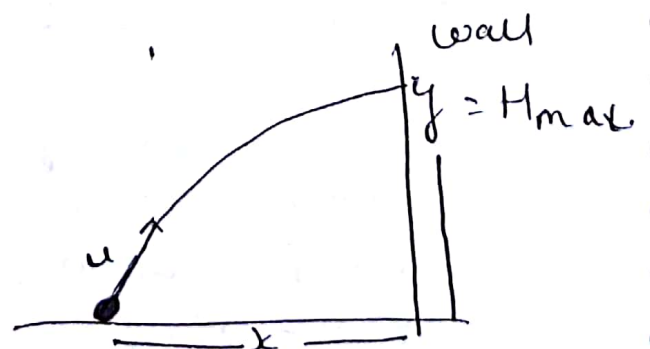
max. height at which projected body strike on wall

$$y = \frac{u^4 - g^2x^2}{2gu^2}$$

$$u^4 - g^2x^2 = 2gyu^2$$

$$u^4 - 2gyu^2 - g^2x^2 = 0$$

$$u^2 = \frac{2gy \pm \sqrt{4g^2y^2 + 4g^2x^2}}{2}$$



$$u^2 = g(y \pm \sqrt{x^2 + y^2})$$

$$u = \sqrt{g(y \pm \sqrt{x^2 + y^2})}$$

$$\tan \alpha = \frac{u^2}{gx} = \frac{y \pm \sqrt{x^2 + y^2}}{x}$$

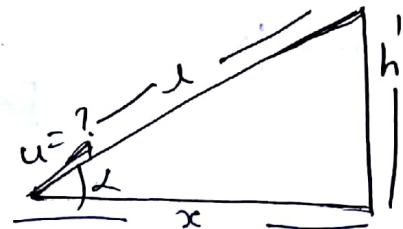
Q//

Required Velocity and angle so that projected body hits to point P.

$$u = \sqrt{g(h \pm \sqrt{x^2 + h^2})}$$

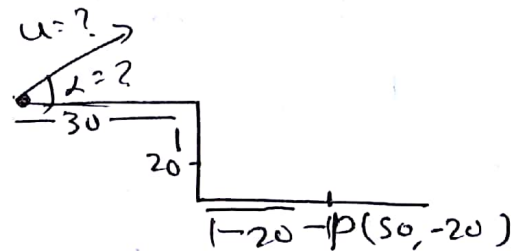
$$u = \sqrt{g(h \pm x)}$$

$$\tan \alpha = \frac{u^2}{gx} \Rightarrow \frac{g(h \pm x)}{g(\sqrt{x^2 + h^2})} \Rightarrow \sqrt{\frac{(L+h)}{(L-h)}}$$



Q//

Required velocity and angle for which projected body hits point P.



$$u = \sqrt{g(y \pm \sqrt{x^2 + y^2})}$$

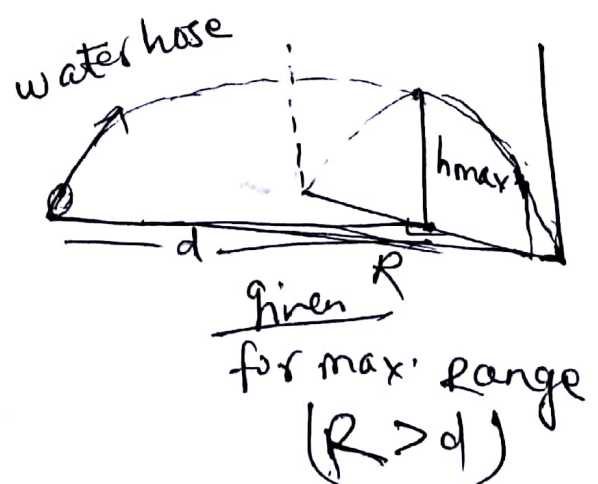
$$u = \sqrt{g(-20 \pm \sqrt{50^2 + 20^2})}$$

$$u = \sqrt{g(-20 \pm 10\sqrt{29})}$$

$$\tan \alpha = \frac{y \pm \sqrt{x^2 + y^2}}{x}$$

Q//

find max. height & width up to which water can spray on the wall.



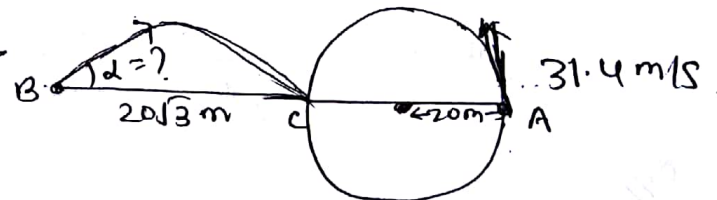
$$H_{\max.} = \frac{u^4 - g^2 x^2}{2gu^2}$$

$$= \frac{g^2 R^2 - g^2 d^2}{2g^2 R}$$

$$H_{\max.} = \frac{R^2 - d^2}{2R}$$

$$\text{width} = \frac{2L}{2\sqrt{R^2 - d^2}}$$

Q. Required angle α for which projected ball B & orbiting body A collide at point C



$$T = \left(n - \frac{1}{2}\right)t \quad n = 1, 2, 3, \dots$$

$$t = \frac{2\pi r}{v}$$

$$t = \frac{2 \times 3.14 \times 20}{31.4} = 4 \text{ second}$$

$$T = \frac{2u \sin \alpha}{g} = \left(n - \frac{1}{2}\right) 4$$

$$R = \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

$$u = \sqrt{\frac{gR}{2 \sin \alpha \cos \alpha}}$$

$$\frac{2u \sin \alpha}{g} \sqrt{\frac{g \times 20\sqrt{3}}{2 \sin \alpha \cos \alpha}} = \left(n - \frac{1}{2}\right) 4$$

Speed and direction of projected body after time t .

$$V \cos \beta = u \cos \alpha \quad \text{--- (1)}$$

$$V \sin \beta = u \sin \alpha - g t$$

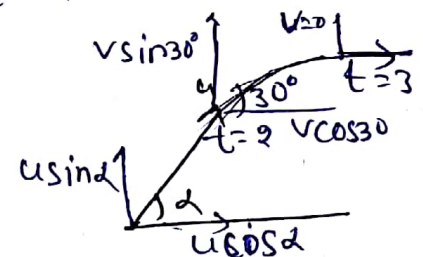
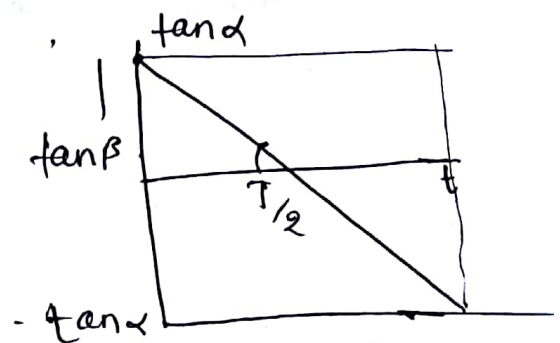
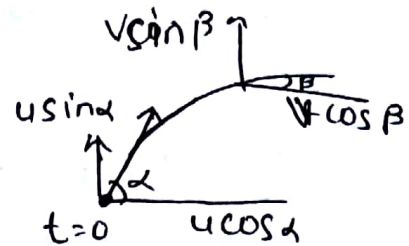
$$V^2 = u^2 + g^2 t^2 - 2 u \sin \alpha g t$$

$$V = \sqrt{u^2 + g^2 t^2 - 2 u \sin \alpha g t}$$

$$\frac{(11)}{(1)} = \tan \beta = \tan \alpha - \left(\frac{g}{u \cos \alpha} \right) t$$

$$y = c - m x$$

$$\tan \theta = \frac{g}{u \cos \alpha}$$



If projected body at angle 30° after 2 second and it become horizontal after 1 more second then find

$$u = ?$$

$$\alpha = ?$$

$$V \sin 30^\circ = u \sin \alpha - g \times 2$$

$$\frac{V}{2} = u \sin \alpha - 20 \quad \text{--- (1)}$$

$$0 = u \sin \alpha - g \times 3$$

$$u \sin \alpha = 30$$

Put in (1)

$$\frac{V}{2} = 30 - 20$$

$$V = 20 \text{ m/s}$$

$$u \cos \alpha = V \cos 30^\circ$$

$$u \cos \alpha = 10\sqrt{3}$$

$$\alpha = 60^\circ$$

$$u = 20\sqrt{3} \text{ m/s}$$

Speed and direction of projected body above height 'h' from point of projection.

$$V \cos \beta = u \cos \alpha \quad \text{--- (I)}$$

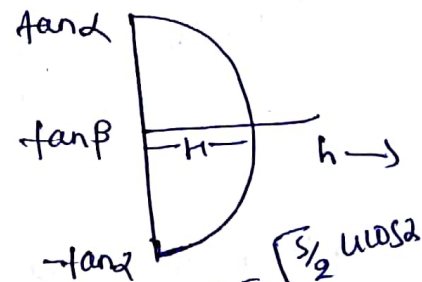
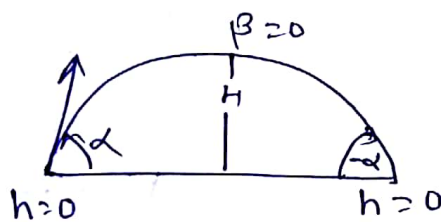
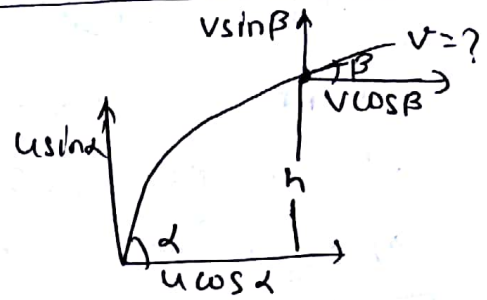
$$V^2 \sin^2 \beta = u^2 \sin^2 \alpha - 2gh \quad \text{--- (II)}$$

squaring (I) and add in (II)

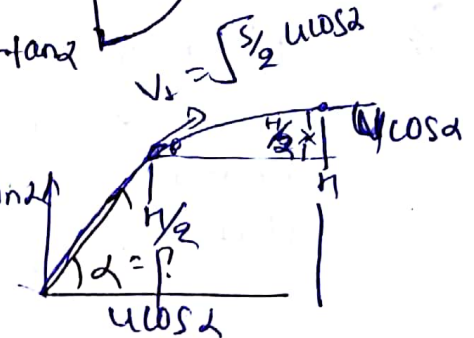
$$V^2 = u^2 - 2gh$$

$$\frac{V^2 \sin^2 \beta}{V^2 \cos^2 \beta} = \frac{u^2 \sin^2 \alpha - 2gh}{u^2 \cos^2 \alpha}$$

$$\tan^2 \beta = \tan^2 \alpha - \left(\frac{2g}{u^2 \cos^2 \alpha} \right) h$$



3// If velocity of projected body at half of its max. height is $\sqrt{\frac{5}{2}} u \sin \alpha$ of its max. height the find angle of projection.



$$V^2 = u^2 - 2gh$$

$$u^2 \cos^2 \alpha = V^2 - 2gh$$

$$u^2 \cos^2 \alpha = \frac{5}{2} u^2 \sin^2 \alpha - g \cdot \frac{u^2 \sin^2 \alpha}{2g}$$

$$\frac{u^2 \sin^2 \alpha}{2} = \frac{3}{2} u^2 \cos^2 \alpha$$

$$\tan^2 \alpha = 3$$

$$\tan \alpha = \sqrt{3}$$

$$\alpha = 60^\circ$$

Time after which projected body become normal to initial direction of projection.

$$\vec{u} \cdot \vec{v} \Rightarrow 0 \text{ means } \vec{u} \perp \vec{v}$$

$$\vec{u} = u \cos \alpha \hat{i} + u \sin \alpha \hat{j}$$

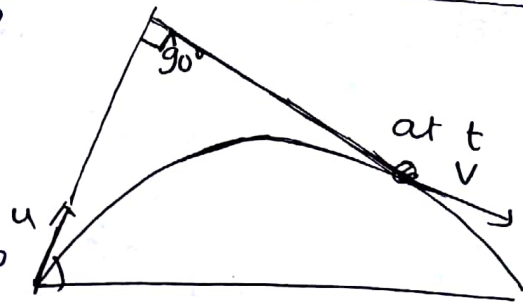
$$\vec{v} = u \cos \alpha \hat{i} + (u \sin \alpha - gt) \hat{j}$$

$$u^2 \cos^2 \alpha + u^2 \sin^2 \alpha - gt u \sin \alpha = 0$$

$$u^2 = gt \cdot u \sin \alpha$$

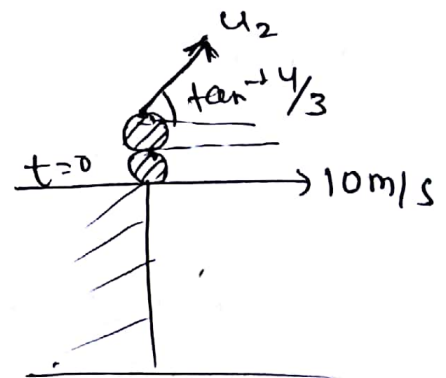
$$t = \frac{u}{g \sin \alpha}$$

$$t \leq \text{time of flight}$$



Q //

Required min. speed u_2 for which projected body become normal to each other & also find time.



After time t

$$\vec{v}_1 = 10 \hat{i} + (-gt) \hat{j}$$

$$\vec{v}_1 = 10 \hat{i} - gt \hat{j}$$

$$\begin{aligned} \vec{v}_2 &= u_2 \cos \alpha \hat{i} + (u_2 \sin \alpha - gt) \hat{j} \\ &= \frac{3}{5} u_2 \hat{i} + \left(\frac{4}{5} u_2 - 10t \right) \hat{j} \end{aligned}$$

$$\vec{v}_1 \cdot \vec{v}_2 = 0$$

$$6u_2 - 8u_2 t + 100t^2 = 0$$

$$50t^2 - 4u_2 t + 3u_2 = 0$$

$t \rightarrow \text{real}$

$$D \geq 0$$

$$16u_2^2 = 4 \times 50 \times 3u_2 \Rightarrow u_2 = 37.5 \text{ m/s}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q time after which projected body become normal to initial direction of projection.

$$t = \frac{u}{g \sin \theta}$$

$$t = \frac{20}{10 \sin 30^\circ} = \frac{4 \text{ second}}{2 \sin 30^\circ}$$

Time of flight = $T =$

$$T = \frac{2 \times 20 \times \sin 30^\circ}{10} \Rightarrow \frac{2 \times 20 \times \frac{1}{2}}{10} = 2 \text{ second}$$

for 90°

$$\frac{u}{g \sin \alpha} \leq T$$

$$1 \leq 2 \sin^2 \alpha$$

$$\sin^2 \alpha \geq \frac{1}{2}$$

$$\sin \alpha \geq \frac{1}{\sqrt{2}}$$

$\alpha \geq 45^\circ$ must for making 90°

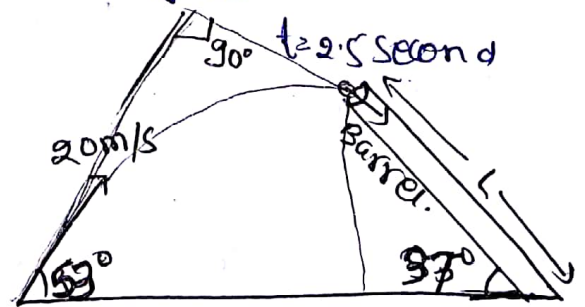
find length "L" for which fired bullet can enter in barrel.

$$t = \frac{u}{g \sin \alpha}$$

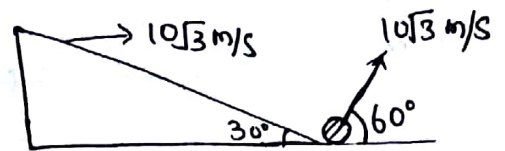
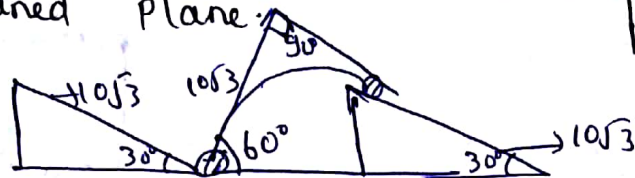
$$t = \frac{20}{10 \times \frac{4}{5}} = 2.5 \text{ second}$$

$$h = u \sin \alpha t - \frac{1}{2} t^2 g \Rightarrow (20 \sin 53^\circ) 2.5 - \frac{1}{2} \times 10 \times (2.5)^2$$

$$L = \frac{h}{\sin 37^\circ}$$



Q// find time after which bullet can strike on the inclined plane.



$$t = \frac{4}{g \sin \alpha} \Rightarrow \frac{10\sqrt{3} \times 2}{10 \times \frac{1}{2}} = 2 \text{ second.}$$

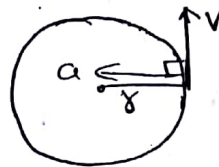
##

RADIUS OF CURVATURE :-

$$a = \frac{v^2}{r}$$

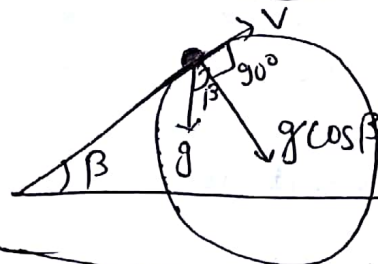
$$\vec{a} \perp \vec{v}$$

$$r = \frac{v^2}{a}$$



$$\omega = \frac{v}{r}$$

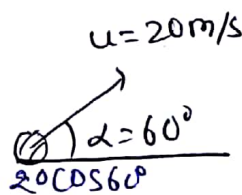
$$\vec{v} \perp \vec{r}$$



$$a = \frac{v^2}{R_c}$$

$$R_c = \frac{v^2}{g \cos \beta}$$

Q//



Radius of curvature:-

① at the time of projection

② at max. height

③ when angle is $\frac{\alpha}{2}$

$$\textcircled{i} \quad R_c = \frac{v^2}{g \cos \alpha} \Rightarrow \frac{(20)^2}{10 \times \cos 60^\circ} = 80 \text{ m}$$

$$\textcircled{ii} \quad R_c = \frac{v^2}{g \cos 0^\circ} \Rightarrow \frac{u^2 \cos^2 \alpha}{g \cos 0^\circ} \Rightarrow \frac{(10)^2}{10} = 10 \text{ m}$$

③

$$20 \cos 60^\circ = v \cos 30^\circ$$

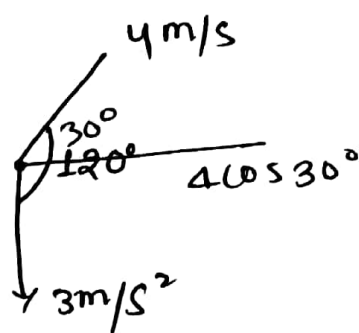
$$v = \frac{20}{\sqrt{3}}$$

$$R_c = \frac{\left(\frac{20}{\sqrt{3}}\right)^2}{10 \times \cos 30^\circ} \Rightarrow \frac{400}{3 \times 10 \times \frac{\sqrt{3}}{2}} = \frac{80}{3\sqrt{3}} \text{ m}$$

Q// Radius of curvature for constant acceleration vector.

$$R_c = \frac{v^2}{a}$$

$$= \frac{\left(4\sqrt{\frac{3}{2}}\right)^2}{3}$$



$$R_c = \frac{2\sqrt{3} \times 2\sqrt{3}}{3} = 4 \text{ second}$$

Q// If velocity of projected body at instant is $\vec{v} = a\hat{i} + b\hat{j}$ then find radius of curvature.

$$v_x = \frac{dx}{dt} = a \rightarrow x = at$$

$$v_y = \frac{dy}{dt} = b$$

$$\int dy = b \cdot a \int t \cdot dt$$

$$y = b \cdot a \frac{t^2}{2}$$

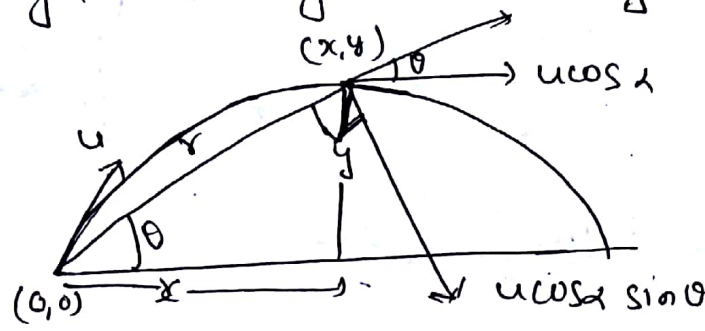
$$y = \frac{b \cdot a}{2} \left(\frac{x^2}{a^2} \right)$$

$$\boxed{y = \frac{bx^2}{2a}} \text{ eq. of Path}$$

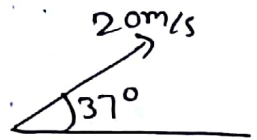
$$R = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

(radius)

ANGULAR VELOCITY :-
find angular velocity at max height

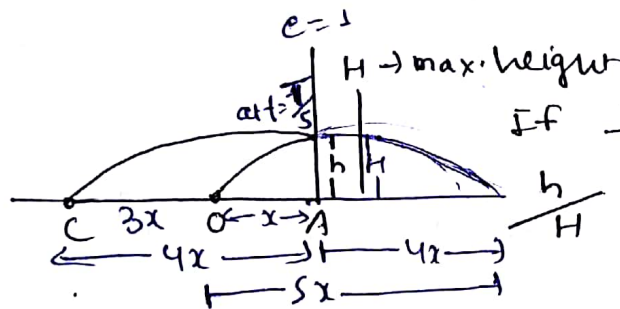


Q//



find angular velocity of projected body at $t = 1$ second

Q//



If $\frac{OA}{OC} = \frac{1}{3}$ then find

$$T = \frac{2u_y}{g}$$

$$H = \frac{u_y^2}{2g}$$

$$h = u_y t + \frac{1}{2} g t^2$$

$$h = u_y \left(\frac{2u_y}{5g} \right) - \frac{1}{2} \left(\frac{2u_y}{5g} \right)^2 g$$

$$h = \frac{u^2}{g} \left[\frac{2}{5} - \frac{4}{2 \times 25} \right]$$

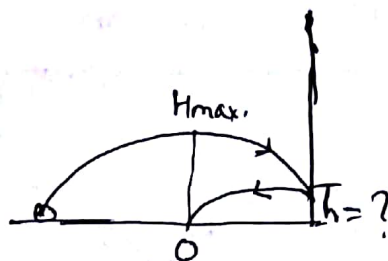
$$h = \frac{u^2}{2g} \left[\frac{4}{5} - \frac{4}{25} \right]$$

$$h = H \left[\frac{20-4}{25} \right]$$

$$\boxed{\frac{h}{H} = \frac{16}{25}}$$

Q//

Q //

 $h = ?$ 

Linear momentum and force :-

at $t=0$

$$\vec{P}_0 = m(u \cos \alpha \hat{i} + u \sin \alpha \hat{j})$$

after time t

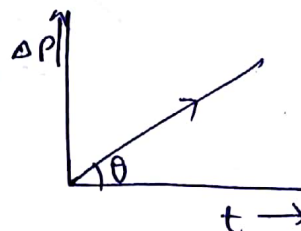
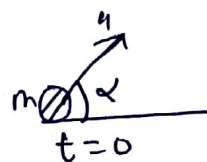
$$\vec{P}_t = m[u \cos \alpha \hat{i} + (u \sin \alpha - gt) \hat{j}]$$

$$\Delta P = \vec{P}_t - \vec{P}_0$$

$$\Delta P = -mgt \hat{j}$$

$$\Delta P = mgt (\downarrow) \text{ downward}$$

$$\text{slope} \Rightarrow \boxed{\tan \theta = mg}$$



$$\boxed{\frac{(\Delta P)_1}{(\Delta P)_2} = \frac{\Delta t_1}{\Delta t_2}}$$

force :-

$$F = \frac{\Delta \vec{P}}{\Delta t} = -mg \hat{j} \Rightarrow mg (\downarrow) \text{ downwards}$$

Angular momentum and torque :-

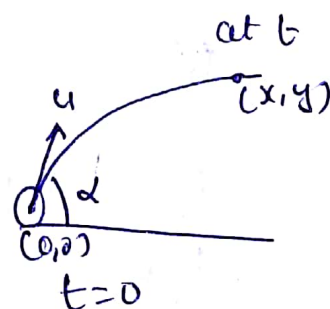
$$\text{at } t \quad \vec{L} = \vec{r} \times \vec{p}$$

$$\vec{r} = u \cos \alpha t \hat{i} + (u \sin \alpha t - \frac{1}{2}gt^2) \hat{j}$$

$$\vec{p} = m[u \cos \alpha \hat{i} + (u \sin \alpha - gt) \hat{j}]$$

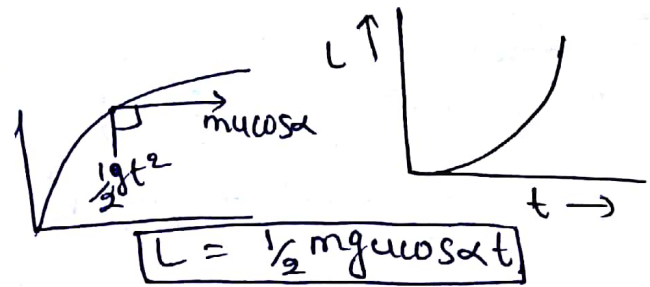
$$\vec{L} = \vec{r} \times \vec{p}$$

$$m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u \cos \alpha t & u \sin \alpha t - \frac{1}{2}gt^2 & 0 \\ u \cos \alpha & u \sin \alpha - gt & 0 \end{vmatrix} \Rightarrow \frac{1}{2} mg u \cos \alpha t^2 (-\hat{k})$$



$$L = \frac{1}{2} m g u \cos \alpha t^2, \text{ Path } \Rightarrow \text{ Parabola}$$

$$\frac{L_1}{L_2} = \frac{t_1^2}{t_2^2}$$

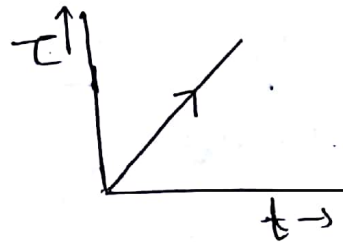


Torque :-

$$\tau = \frac{dL}{dt}$$

$$\vec{\tau} = (m g u \cos \alpha) t (-\hat{k})$$

$$\tau \propto t$$



Energy :-

$$K = K.E. = \frac{1}{2} m u^2$$

$$U = P.E. = 0$$

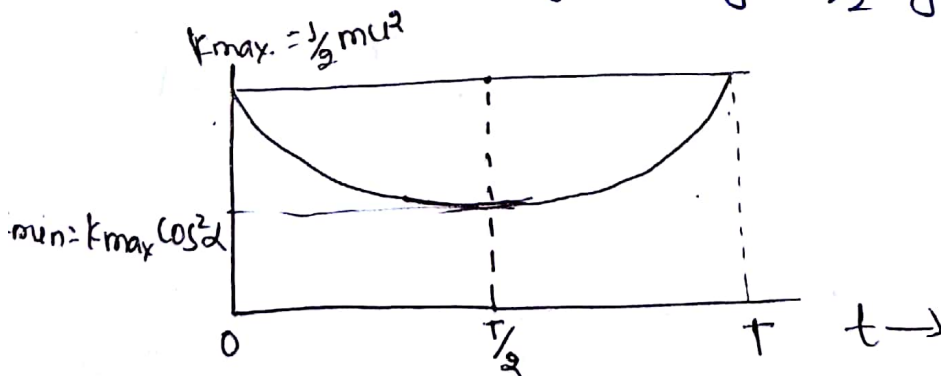
$$T.E. = \frac{1}{2} m u^2$$

at time t

$$K.E. = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m (u^2 + g^2 t^2 - 2 u \sin \alpha g t)$$

$$= \frac{1}{2} m u^2 - (m u \sin \alpha g t - \frac{1}{2} m g^2 t^2) \Rightarrow \text{Parabola}$$



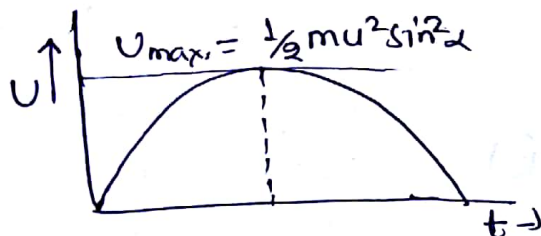
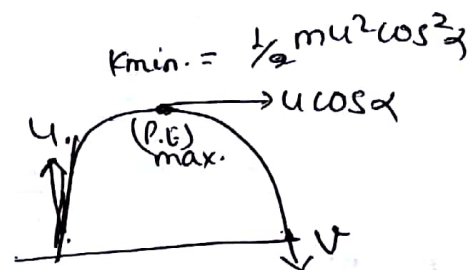
for projectile K.E. can never be zero

(2) P.E. at time t

$$U = mg(us \sin \alpha t - \frac{1}{2}gt^2)$$

$$= mgus \sin \alpha t - \frac{1}{2}mg^2t^2$$

$$=$$



at max. height

K.E.	$= \frac{1}{2}mu^2 \cos^2 \alpha$
P.E.	$= \frac{1}{2}mu^2 \sin^2 \alpha$

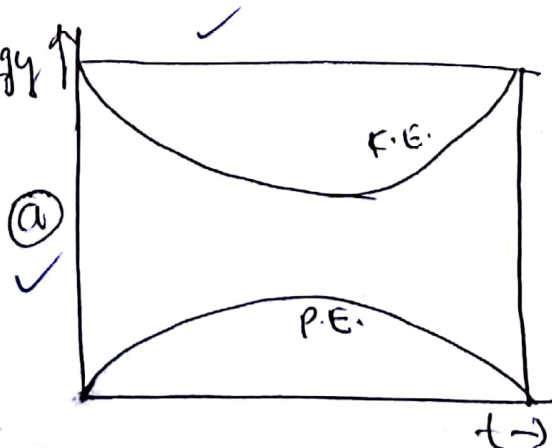
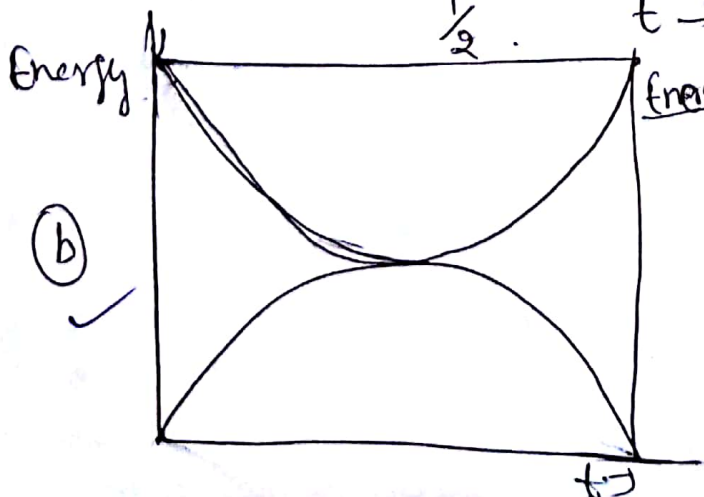
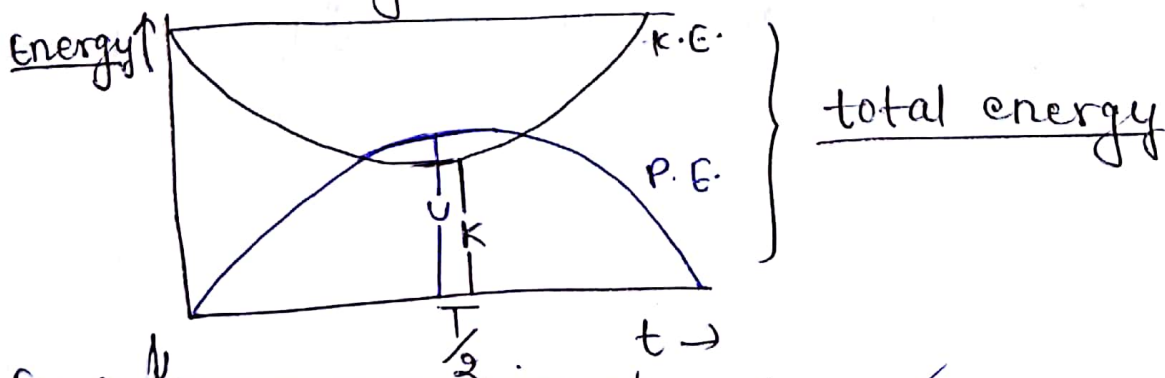
Q.// At max. if P.E. is 3 times of K.E. then find angle of projection.

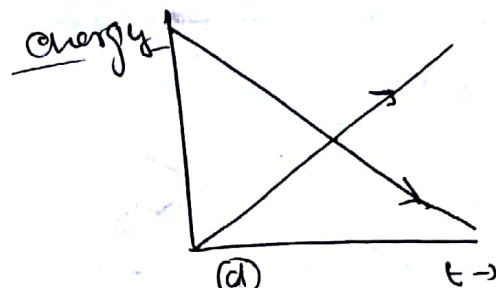
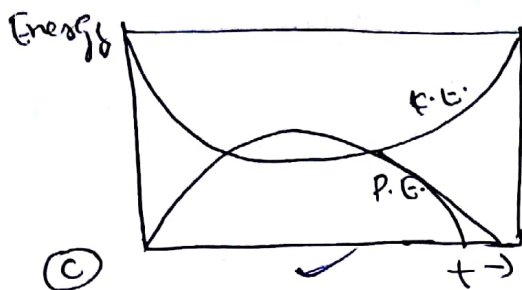
$$\frac{P.E.}{K.E.} = \tan^2 \alpha$$

$$\tan \alpha = \sqrt{\frac{U}{K}} = \sqrt{\frac{3K}{K}} = \sqrt{3}$$

$$\alpha = 60^\circ \text{ Ans.}$$

F Total energy





Q.// which one is a suitable graph for projectile motion?
a, b, c

$$\tan \alpha = \sqrt{\frac{P.E.}{K.E.}}$$

(in) graph (a) $P.E. < K.E.$

$$\tan \alpha < 1$$

$$\alpha < 45^\circ$$

in graph (b) $P.E. = K.E.$

$$\tan \alpha = 1$$

$$\alpha = 45^\circ$$

in graph (c) $P.E. > K.E.$

$$\tan \alpha > 1$$

$$\alpha > 45^\circ$$

Energy at height "h" :-

$$\Rightarrow K.E. = \frac{1}{2} m v^2$$

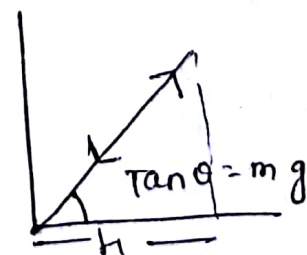
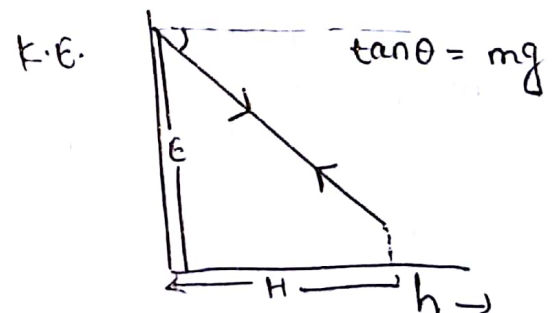
$$= \frac{1}{2} m (u^2 - 2gh)$$

$$= \frac{1}{2} m u^2 - mgh$$

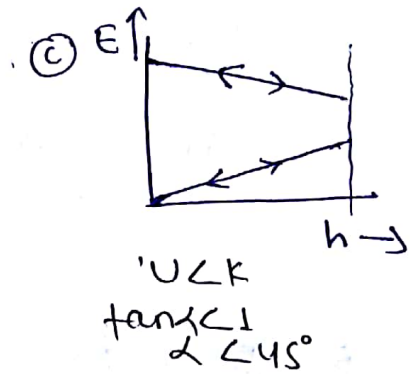
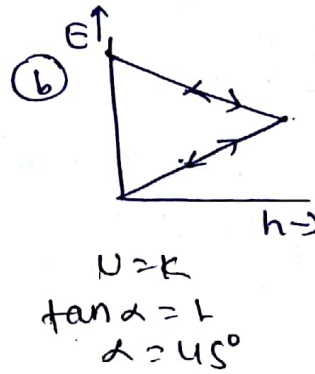
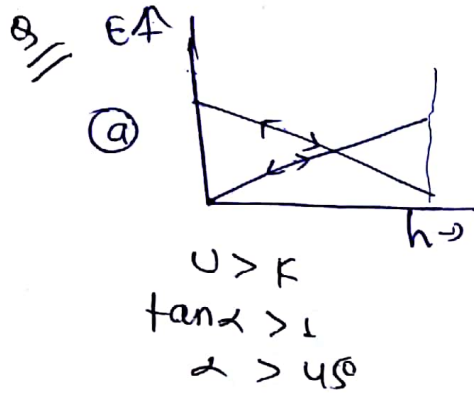
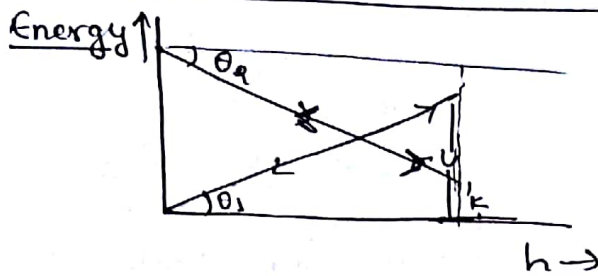
$$K = E - (mgh)$$

$$\Rightarrow P.E. = mgh$$

$$v = \sqrt{2gh}$$

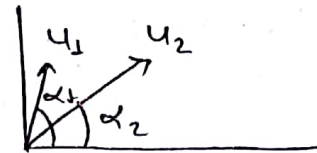
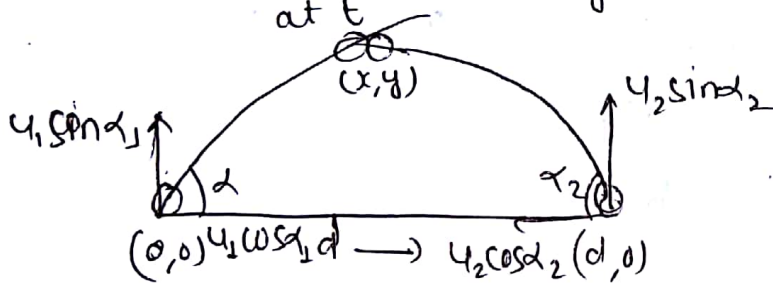


graph of K.E. & P.E. with height :-



When and Where Projected body will collide for Simultaneous projection :-

$\Rightarrow$ from same height



along x-axis

$$t = \frac{d_{rel}}{v_{rel}} = \frac{d}{u_1 \cos \alpha_1 + u_2 \cos \alpha_2}$$

where

$$x = u_1 \cos \alpha_1 t$$

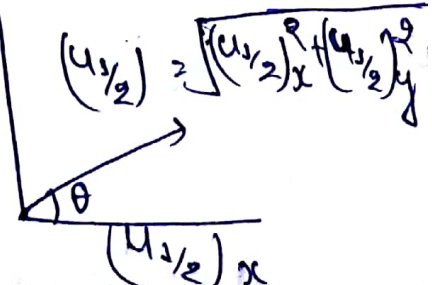
$$y = u_1 \sin \alpha_1 t - \frac{1}{2} g t^2$$

Path of Projected body ① from Projectile ②

$$(u_{1/2})_x = u_1 \cos \alpha_1 - u_2 \cos \alpha_2 \quad (u_{1/2})_y$$

$$(u_{1/2})_y = u_1 \sin \alpha_1 - u_2 \sin \alpha_2$$

$$\tan \theta = \frac{(u_{1/2})_y}{(u_{1/2})_x} \Rightarrow \frac{(u_{1/2})_y t}{(u_{1/2})_x t} \Rightarrow \frac{y}{x}$$



$$y = mx$$

straight line ✓

Q//:- If collision is possible then find when & where collide

for collision

$$t = \frac{400}{20(1+\sqrt{3})} = \frac{20}{\sqrt{3}+1} = t > 6 \text{ second}$$

flight time

$$T = \frac{2 \times 20}{10} = 4 \text{ second}$$

$t \geq T$ not possible

for collision $t \leq T$ it is possible condition

Q// find time interval b/w two body for collision

$$x_1 = x_2$$

$$5\sqrt{3}(t-\Delta t) = \frac{5\sqrt{3}}{2}t$$

$$2(t-\Delta t) = t$$

$$y_1 = y_2$$

$$\frac{1}{2}g(t-\Delta t)^2 = -5\sqrt{3} \sin 60^\circ t + \frac{1}{2}gt^2 \quad (\text{put } t = 2(t-\Delta t))$$

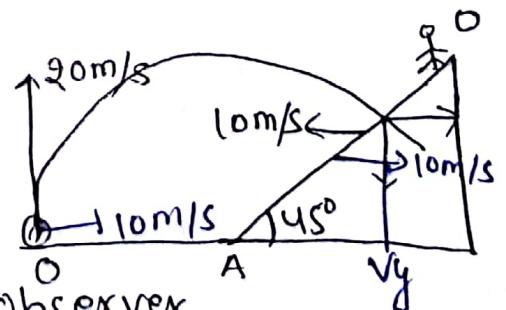
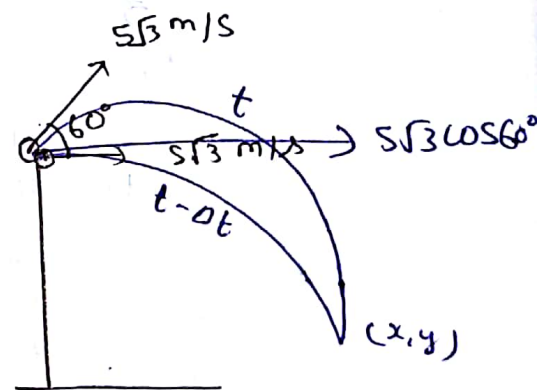
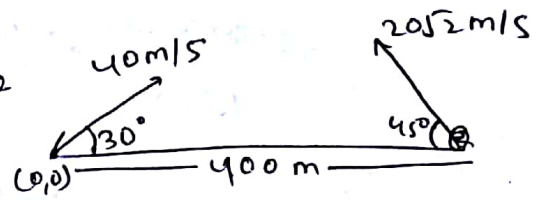
$$\Delta t = 1 \text{ second}$$

Q// find separation b/w O and A when projected body collide on inclined plane at 90° .

velocity of projected body for observer

$$\vec{v} = 20\hat{j} - (-10\hat{i}) \Rightarrow 10\hat{i} + 20\hat{j}$$

$$\tan 45^\circ = \frac{v_x}{v_y} \Rightarrow v_y = 10 \text{ m/s}$$



$$R = 10 \times 3 = 30 \text{ m}$$

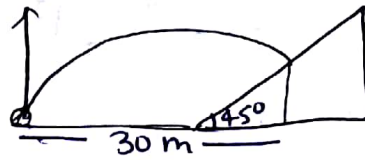
$$Y = u_y t - \frac{1}{2} g t^2$$

$$= 20 \times 3 - \frac{1}{2} \times 10 \times 3^2$$

$$= 60 - 45 = 15 \text{ m}$$

$$OA = \tan 45^\circ = \frac{15}{x}$$

$$OA = 15 \text{ m}$$



find height $h = ?$

$$\tan 53^\circ = \frac{V_x}{V_y}$$

$$\frac{4}{3} = \frac{12}{V_y}$$

$$V_y = 9 \text{ m/s}$$

$$v = u - gt$$

$$-9 = 11 - 10t$$

$$t = 2.5 \text{ second}$$

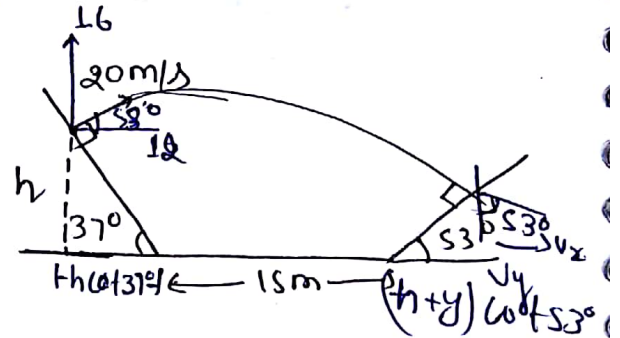
$$30 = h \times \frac{4}{3} + 15 + \frac{3}{4}(y+h)$$

what is $\omega_{\min}$.

$$R \cos \theta + \frac{V^2 \sin^2 \theta}{2g} = H$$

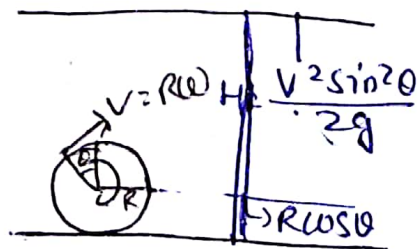
$$\frac{dH}{d\theta} = 0$$

Ans.



$$y = 16 \times 2.5 - \frac{1}{2} \times 10 \times (2.5)^2 \quad \text{--- (1)}$$

$$12 \times t = 12 \times 2.5 = 30 \text{ m}$$



Motion along Inclined Plane:-

① Flight time:-

$$(T) = \frac{2u \sin(\alpha - \beta)}{g \cos \beta}$$

② Max. height:-

$$(H) = \frac{u^2 \sin^2(\alpha - \beta)}{2g \cos \beta}$$

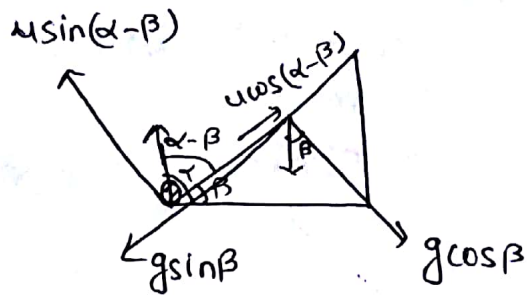
③ Range :-

$$(R) = R = u \cos(\alpha - \beta) T - \frac{1}{2} g \sin \beta T^2$$

$$= \frac{u^2 [\sin(2\alpha - \beta) - \sin \beta]}{g \cos^2 \beta}$$

for $R_{\max}$ $= 2\alpha - \beta = 90^\circ$

$$R_{\max} = \frac{u^2}{g(1 + \sin \beta)}$$



case - II

① Flight time

$$(T) = \frac{2u \sin(\alpha + \beta)}{g \cos \beta}$$

$\alpha = \oplus ve$
 $\beta = \ominus ve$

② max. height

$$(H) = \frac{u^2 \sin^2(\alpha + \beta)}{2g \cos \beta}$$

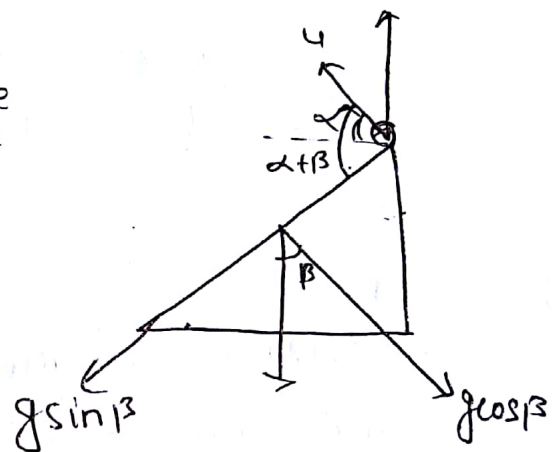
③

Range

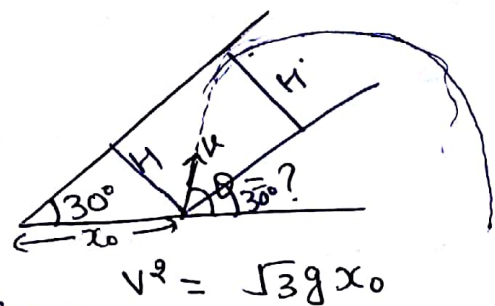
$$(R) = \frac{u^2 [\sin(2\alpha + \beta) + \sin \beta]}{g \cos^2 \beta}$$

$\alpha = 45^\circ - \frac{\beta}{2}$

$R_{\max} \rightarrow \frac{u^2}{g(1 - \sin \beta)}$



Q// find angle θ for which projected body passes through inclined plane & strike on earth



$$H = x_0 \sin 30^\circ$$

$$V^2 = \sqrt{3gx_0}$$

$$H = \frac{u^2 \sin^2(\theta - \phi)}{2g \cos \phi}$$

$$= \frac{\sqrt{3gx_0} \sin^2(\theta - 30^\circ)}{2g \cos 30^\circ}$$

$$\cancel{x_0} \sin 30^\circ = \frac{\sqrt{3} \cancel{g} \cancel{x_0} \sin^2(\theta - 30^\circ)}{2g \cos 30^\circ}$$

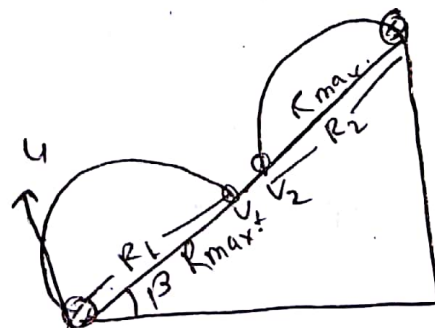
$$\frac{1}{2} = \frac{\sqrt{3} \sin^2(\theta - 30^\circ)}{2 \times \frac{\sqrt{3}}{2}}$$

$$\sin^2(\theta - 30^\circ) = \frac{1}{2}$$

$$\theta - 30^\circ = 45^\circ$$

$$\theta = 75^\circ \text{ Ans.}$$

Q// find velocity of projection, angle and max. range on horizontal plane



Q, max. range after striking the earth is

$$\textcircled{i} \frac{v_1 + v_2}{2}$$

$$\textcircled{ii} \frac{2v_1 v_2}{v_1 + v_2}$$

$$\textcircled{iii} u = \sqrt{v_1 v_2}$$

$$\textcircled{iv} \text{ none}$$

$$R_1 = \frac{u^2}{g(1 + \sin \beta)} \quad \textcircled{i}$$

$$R_2 = \frac{u^2}{g(1 - \sin \beta)} \quad \textcircled{ii}$$

$$R = \frac{u^2}{g} \quad \textcircled{iii}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{g(1+\sin\beta)}{u^2} + \frac{g(1-\sin\beta)}{u^2}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$$

$$R = \frac{2(R_1 R_2)}{(R_1 + R_2)}$$

(2)

Horizontal plane

$$R = \frac{u^2}{g}$$

$$u = \sqrt{gR}$$

$$= \sqrt{\frac{2gR_1 R_2}{(R_1 + R_2)}}$$

$$\frac{R_1}{R_2} = \frac{1 - \sin\beta}{1 + \sin\beta} \Rightarrow$$

$$\frac{R_2 - R_1}{R_2 + R_1} = \sin\beta$$

(3)

$$v_1^2 = u^2 - 2gR_{\max}\sin\beta$$

$$u^2 - 2g\sin\beta \left[\frac{u^2}{g(1+\sin\beta)} \right]$$

$$u^2 \left[1 - \frac{2\sin\beta}{1+\sin\beta} \right] \Rightarrow u^2 \left[\frac{1-\sin\beta}{1+\sin\beta} \right]$$

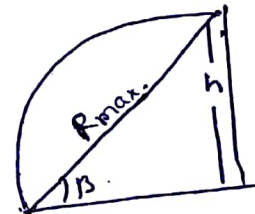
$$v_1 = u \sqrt{\frac{1-\sin\beta}{1+\sin\beta}}$$

$$v_2 = u \sqrt{\frac{1+\sin\beta}{1-\sin\beta}}$$

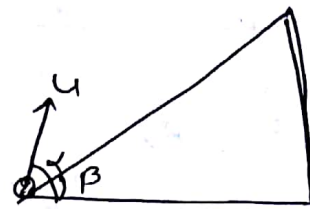
$$\frac{v_1 v_2}{\cancel{u^2}} = u^2$$

$$u = \sqrt{v_1 v_2}$$

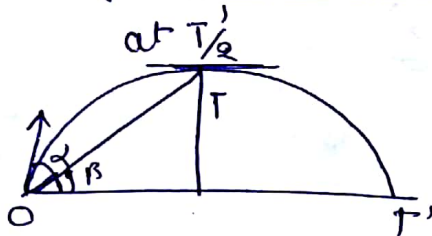
$$\frac{v_1}{v_2} = \frac{1-\sin\beta}{1+\sin\beta} = \frac{R_1}{R_2}$$



- Q. find angle α for which
Projected body strike
- (i) horizontally on plane
 - (ii) Normally to plane.



(i) motion for horizontal plane:-



flight time =
 $T = T'/2$

$$\frac{2u \sin(\alpha - \beta)}{g \cos \beta} = \frac{u \sin \alpha}{g}$$

$$2 \sin(\alpha - \beta) = \sin \alpha \cos \beta$$

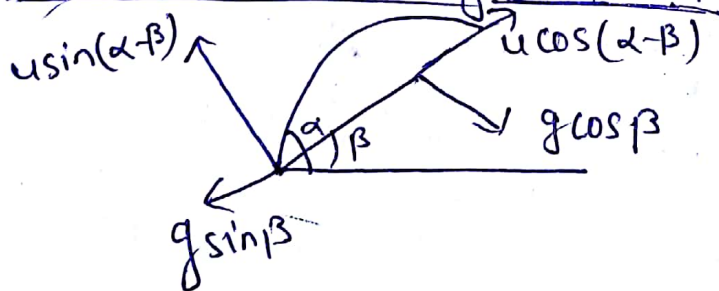
$$2 \sin \alpha \cos \beta - 2 \cos \alpha \sin \beta = \sin \alpha \cos \beta$$

$$\sin \alpha \cos \beta = 2 \cos \alpha \sin \beta$$

$$\tan \alpha = 2 \tan \beta$$

$$\alpha = \tan^{-1}(2 \tan \beta) \text{ Ans.}$$

(ii) motion for Normally to plane:-



At the collision time
Velocity || to Plane = 0

$$u \cos(\alpha - \beta) - g \sin \beta t = 0$$

$$t = \frac{u \cos(\alpha - \beta)}{g \sin \beta}$$

$$T = t$$

$$\frac{2u \sin(\alpha - \beta)}{g \cos \beta} = \frac{u \cos(\alpha - \beta)}{g \sin \beta}$$

$$2 \tan(\alpha - \beta) = \cot \beta$$

Q11

find position from vertical ball where projected ball strike on earth.

$$h = -ut + \frac{1}{2}gt^2$$

$$1.05 = -10t + 5t^2$$

$$5t^2 - 10t - 1.05 = 0$$

$$t = 2.1 \text{ second}$$

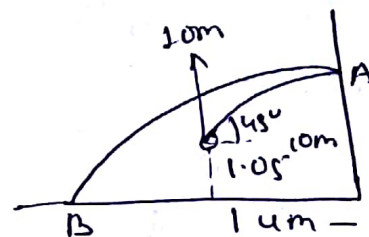
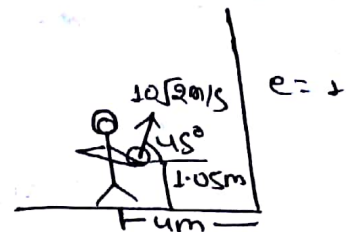
$$t_{0 \rightarrow A} = \frac{4}{10}$$

$$t_{0 \rightarrow A} = 0.4 \text{ second}$$

$$t_{0 \rightarrow B} = 2.1 - 0.4 = 1.7 \text{ second}$$

$$S = Vt_{0 \rightarrow B}$$

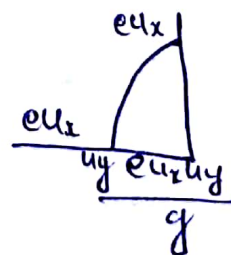
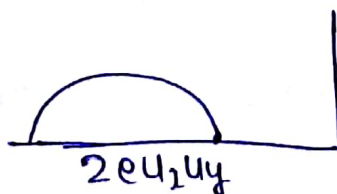
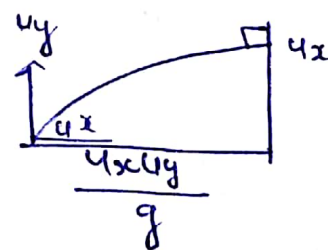
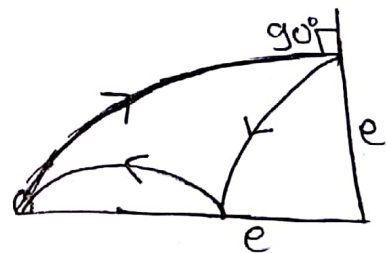
$$= 10 \times 1.7 = 17 \text{ m} \quad \text{Ans}$$



Q12

find value of e for which projected body comes to its initial position.

- ① $\frac{1}{3}$ ② 1 ③ $\frac{1}{4}$ ④ none



$$\frac{u_y}{g} = \frac{2e^2 u_x u_y}{g} + \frac{e u_x u_y}{g}$$

$$2e^2 + e - 1 = 0$$

$$\Rightarrow \frac{-1 \pm \sqrt{1+8}}{4}$$

$$= \frac{-1+3}{4} = \frac{1}{2}$$

Q.11 for same range if flight time of projected body is 3 second and 4 second then find max. range.

Same Range

$$\alpha, \quad 90 - \alpha = \beta$$

$$T_1 = \frac{2u \sin \alpha}{g}$$

$$T_2 = \frac{2u \cos \alpha}{g}$$

$$u \sin \alpha = \frac{g T_1}{2} \quad \text{--- (1)}$$

$$u \cos \alpha = \frac{g T_2}{2} \quad \text{--- (2)}$$

Solve (1) & (2)

$$u = \frac{g}{2} \sqrt{T_1^2 + T_2^2}$$

$$\boxed{\tan \alpha = \frac{T_1}{T_2}}$$

$$\tan \beta = \tan(90 - \alpha)$$

$$\boxed{\tan \beta = \frac{T_2}{T_1}}$$

$$H_1 = \frac{g T_1^2}{8}$$

$$H_2 = \frac{g T_2^2}{8}$$

$$R = \frac{g}{2} T_1 T_2$$

$$R_{\max.} = \frac{u^2}{g} \Rightarrow \frac{g}{4} (T_1^2 + T_2^2)$$

$$\Rightarrow \frac{g}{4} 25 \Rightarrow \frac{250}{4} = \boxed{62.5 \text{ m}} R_{\max.}$$

for same range if maximum height of projected body is 4m and 9m then find max. range

$$H_1 = \frac{u^2 \sin^2 \alpha}{2g}$$

$$H_2 = \frac{u^2 \cos^2 \alpha}{2g}$$

$$u^2 \sin^2 \alpha = 2g H_1$$

$$u^2 \cos^2 \alpha = 2g H_2$$

$$u = \sqrt{2g(H_1 + H_2)}$$

$$\tan \alpha = \sqrt{\frac{H_1}{H_2}}, \quad \tan \beta = \sqrt{\frac{H_2}{H_1}}$$

$$T_1 = \sqrt{\frac{8H_1}{g}}, \quad T_2 = \sqrt{\frac{8H_2}{g}}$$

$$R_{\max.} = \frac{2(u \sin \alpha)(u \cos \alpha)}{g} = R = 4 \sqrt{H_1 H_2}$$

$$R_{\max.} = \frac{u^2}{g} = 2(H_1 + H_2) = 2(9 + 4) = 26 \text{ m}$$

Q//

find $\frac{R_1}{R_2}$

